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Penalty Function Method For Solving Fuzzy Nonlinear Programming Problem

[1]A.F.Jameel and [2]Radhi.A.Z

[1]School of Mathematical Sciences, 11800 USM, University Science Malaysia, Penang, Malaysia
[2]Department of Mathematics, College of Science, the University of AlMustansiriyah Iraq Baghdad

Abstract: **In this work, the fuzzy nonlinear programming problem (FNLPP) has been developed and their result have also discussed. The numerical solutions of crisp problems and have been compared and the fuzzy solution and its effectiveness have also been presented and discussed. The penalty function method has been developed and mixed with Nelder and Mend's algorithm of direct optimization problem solutionhave been used together to solve this FNLPP.**

Keyword: **Fuzzy set theory, fuzzy numbers, decision making, nonlinear programming, Nelder and Mend's algorithm, penalty function method.**

I. Introduction

Fuzzy nonlinear programming problem (FNLPP) is useful in solving problems which are difficult, impossible to solve due to the imprecise, subjective nature of the problem formulation or have an accurate solution. In this paper, we will discuss the concepts of fuzzy decision making introduced by [2] and the maximum decision [20] that is used in NLPP to find the optimal decision (solution). This decision making was used in fuzzy linear and nonlinear programming problems [1],[8], [9] and [15]. Furthermore, these problemshave fuzzy objective function and fuzzy variables in the constraints [5], [10], [11] and[17] where the fuzzy left and right hand side coefficients on constraints [18]. In addition, the fuzzy NLPP is used in quadratic programming [6], [12] and [16] which hasa fuzzy multi objective function and fuzzy parameters on constraints so in our NLPP that have fuzzy properties on.However, the fuzzy nonlinear programming problem is not just an alternative or even a superior way of analyzing a given problem, it's useful in solving problems in which difficult or impossible to use due to the inherent qualitative imprecise or subjective nature of the problem formulation or to have an accurate solution.The outline of this study is as follows: In section two we introduce some important definitions that are useful in our problem. Section3 we state the general nonlinear programming problem in fuzzy environment by transforming the crisp problem into the fuzzy problem. Section 4 we present and develop the regular penalty function method and mixed it with Nelder and Mend's algorithm in order to solve FNLPP. Finally, in section 5, we show the efficiency of our study by present numerical example involving FNLPP.

II. Primetimes

1. Fuzzy Set [20]:

If x is a collection of objects denoted generally by X, then a fuzzy set $\tilde{A}$ in X is a set of order pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$,where $\mu_{\tilde{A}}(x) : x \longrightarrow [0, 1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$ which maps x to the membership range M (when M contains only the two points 0 and 1), $\tilde{A}$ is a nonfuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set. The range of membership function is a subset of the non-negative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

2. Fuzzy Numbers with Linear Member Ship Function [3]:

The function L: X $\longrightarrow$[0, 1] is a function with two parameters defined as:

$$\mathbf{L}(\mathbf{x};\alpha,\beta)=\begin{cases}1, & if\ \ x<\alpha \\ \dfrac{\alpha+\beta-x}{\beta}, & if\ \ \alpha\le x\le\alpha+\beta \\ 0, & if\ \ x>\beta\end{cases}$$

Where L is called the trapezoidal linear membership function.

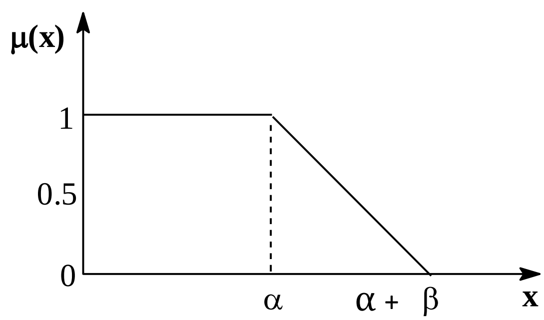

Fig. 1 L-Function.

3. Fuzzy Decision Making [2]:

Assume that we are given a fuzzy goal (fuzzy objective function) $\tilde{G}$ and fuzzy constraints $\tilde{C}$ in a space of alternatives X. The $\tilde{G}$ and $\tilde{C}$ combine to form a decision, $\tilde{D}$, which is a fuzzy set resulting from intersection of $\tilde{G}$ and $\tilde{C}$. In symbols, $\tilde{D}\cap\tilde{C}$ is, correspondingly, the membership function of $\tilde{D}$ can be defined as:

$\mu_{\tilde{D}}=$ Min { $\mu_{\tilde{G}}$, $\mu_{\tilde{C}}$ }.More generally, suppose that we have n goals $\tilde{G}_1,\tilde{G}_2,\ldots,\tilde{G}_n$ and m constraints $\tilde{C}_1,\tilde{C}_2,\ldots,\tilde{C}_m$. The, the resultant decision is defined as:

$\tilde{D}=\tilde{G}_1\cap\tilde{G}_2\cap\ldots\cap\tilde{G}_n\cap\tilde{C}_1\cap\tilde{C}_2\cap\ldots\cap\tilde{C}_m$ and correspondingly:

$$\mu_{\tilde{D}}=\text{Min}\,\{\min\{\mu_{\tilde{G}_1},\mu_{\tilde{G}_2},\ldots,\mu_{\tilde{G}_n}\},\min\{\mu_{\tilde{C}_1},\mu_{\tilde{C}_2},\ldots,\mu_{\tilde{C}_m}\}\}$$

$$=\text{Min}\,\{\mu_{\tilde{G}_1},\mu_{\tilde{G}_2},\ldots,\mu_{\tilde{G}_n},\mu_{\tilde{C}_1},\mu_{\tilde{C}_2},\ldots,\mu_{\tilde{C}_m}\}=\text{Min}\,\{\mu_{\tilde{G}_j},\mu_{\tilde{C}_j}\}$$

for j = 1, 2,…, n and i = 1, 2, …, m.

4. Maximum Decision Maker [20]:

If the decision-maker wants to have “crisp” decision proposal, it seems appropriate to suggest to him the divided which has the highest degree of membership in the fuzzy set “decision”. Let us call this the maximizing decision, defined by:

$$x_{max} = \underset{x}{\mathrm{Max}}\, M_{\tilde{D}}(x) = \underset{x}{\mathrm{Max}}\, \mathrm{Min}\left\{\mu_{\tilde{D}_j}(x), \mu_{\tilde{C}_i}(x)\right\},$$

where $\tilde{D}_j$ and $\tilde{C}_i$ are in the definition (3) for i = 1, 2, …, m; j =1, 2, …, n.

III. Penalty Function Method

There are survival methods used in fuzzy reliability problems, such as a novel approach for solving unconstrained model mechanical structure [7], also the numerical integration algorithm in the fuzzy general strength model [13]. The penalty method [19] belongs to the first attempts to solve constrained optimization problems satisfactorily. The basic idea is to construct a sequence of unconstrained optimization problems and solve them by standard minimize or maximize of the unconstrained problems converge to the solution of the constrained one. To simplify the notation, we consider the following two NLPP:

$$\begin{aligned} &(A)\, \mathrm{Min/Max}\, f(x) \\ &\quad \text{Subject to:} \\ &\qquad g_i(x) \geq (\leq) 0, \\ &i = 1, 2, \ldots, m \text{ and } x \geq 0, \forall\, x \in R^n \end{aligned} \tag{1}$$

$$\begin{aligned} &(B)\, \mathrm{Min/Max}\, f(x) \\ &\quad \text{Subject to:} \\ &\qquad g_i(x) = 0, \\ &i = 1, 2, \ldots, m \text{ and } x \geq 0, \forall\, x \in R^n \end{aligned} \tag{2}$$

To construct the unconstrained problems, so-called penalty terms are added to the objective function which penalizes f(x) whenever the feasibility region is left. A factor σ_k controls the degree of penalizing f(x).

Proceeding from a sequence $\{\sigma_k\}$ with $\sigma_k \longrightarrow \infty$ for k = 0, 1… penalty function can be defined by [4]:

1. Min/Max $\varphi(x,\sigma) = f(x) + \frac{1}{2}\sigma_k \sum_{i=1}^{m} (\min(0, g_i(x)))^2$ (3)

for problem (1).

2. Min/Max $\varphi(x, \sigma) = f(x) + \frac{1}{2}\sigma_k \sum_{i=1}^{m} (g_i(x))^2$ (4)

for problem (2)

The unconstrained nonlinear programming problems are solved by any standard technique, e.g.**,** Nelder and Mead [14] method combined with a line search. However, the line search must be performed quite accurately due to this step, narrow valleys created by the penalty terms, respectively. The technique of solving a sequence of minimization (maximization) problems of by using a penalty function method is as follows:

1. Choose a sequence $\{\sigma_k\} \longrightarrow \infty$.
2. For each σ_k finding a local minimizer (maximizer) $x(\sigma_k)$ say, $\underset{x}{\min}(\underset{x}{\max})\, \varphi(x, \sigma_k)$ .By any steeple optimization method.

3. Stop when the penalty terms $\frac{1}{2}\sigma_{ok}$, or $\frac{1}{2}\sigma_k \sum_{i=1}^{m}(g_i(x))^2$ is zero and the constraints satisfies the solution at once when the penalty terms is zero.
4. The convergence of the solution of (NLPP) is using the penalty function method and its properties can be found in [4].
5. Use this formula (3-4) in Nelderand Mead's [14] algorithm for the direct solution of the optimization problem.

IV. Fuzzy Nonlinear Programming Problem

Consider the crisp NLPP below:

Min/Max f(x)

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \leq (\geq)\ b_i,\ i = 1, 2\ldots m;\ j = 1, 2\ldots n \qquad (5)$$

For all $x_j \geq 0$ and $x_j \in R^n$.

Now, the fuzzy version for problem (1) is as follows:

$$\widetilde{\text{Min}}/\widetilde{\text{Max}}\ f(x_j)$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq (\geq)\ \tilde{b}_i,\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \qquad (6)$$

For all $x_j \geq 0$ and $x_j \in R^n$.

Note that, the i-th constraint of problem (2) is called fuzzy technological constraint with fuzzy technological coefficients and fuzzy right hand side numbers, where $\tilde{a}_{ij}$ is the fuzzy technological coefficient and $\tilde{b}_i$ is the fuzzy right hand side numbers, where i=1, 2, …, m and j=1, 2, …, n.This problem can be solved using fuzzy decision making properties by using definition (3) and (4) in section II as follows:

1. To fuzzify the objective function, calculate the lower and the upper bounds z_λ and z_u receptively of the optimal values are obtained by solving the crisp NLPP as follows:

z_1= Min/Max $f(x_j)$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \geq (\leq)\ b_i \qquad (7)$$

$\forall\ x_j \geq 0,\ x_j \in R^n$ and i = 1, 2, …, m; j =1, 2, …, n.

z_2= Min/Max $f(x_j)$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \geq (\leq)\, b_i + p_i \qquad (8)$$

$\forall\ x_j \geq 0,\ x_j \in R^n$ and i = 1, 2, …, m; j =1, 2, …, n.

z_3= Min/Max $f(x_j)$

Subject to:

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \geq (\leq)\, b_i + p_i \qquad (9)$$

$\forall\ x_j \geq 0,\ x_j \in R^n$ and i = 1, 2, …, m; j =1, 2, …, n.

and

z_4= Min/Max $f(x_j)$

Subject to:

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \geq (\leq)\, b_i \qquad (10)$$

$\forall\ x_j \geq 0,\ x_j \in R^n$ and i = 1, 2, …, m; j =1, 2, …, n.

Where, $p_i, d_{ij} > 0$ are any chosen constants represented the limits of b_i and a_{ij}.Whenever the objective function takes the value between z_1, z_2, z_3 and z_4 by letting z_λ= min (z_1, z_2, z_3, z_4) and z_u= max (z_1, z_2, z_3, z_4). Let $\tilde{M}$ be the fuzzy set representing the objective function $f(x_j)$, such that:

$$\tilde{M} = \{(x, \mu_{\tilde{M}}(x)) \mid x \in R^n\}$$

Where:

$$\mu_{\tilde{M}}(x) = \begin{cases} 1, & if \quad z_u < f(x_j) \\ \dfrac{f(x_j) - z_\ell}{z_u - z_\ell}, & if \quad z_\ell \leq f(x_j) \leq z_u \\ 0, & if \quad f(x_j) < z_\ell \end{cases}$$

This represents the satisfaction of the aspiration level of the objective.Note that, p_i and d_{ij} are any chosen constant represented the limits of b_i and d_{ij} and can be found as follows:

$$\tilde{a}_{ij}(x) = \{(x, \mu_{\tilde{a}_{ij}}(x)) \mid x \in R\}$$

Where:

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 1, & if \quad x < a_{ij} \\ \dfrac{a_{ij} + d_{ij} - x}{d_{ij}}, & if \quad a_{ij} \leq x \leq a_{ij} + d_{ij} \\ 0, & if \quad x > a_{ij} + d_{ij} \end{cases}$$

See Fig.1

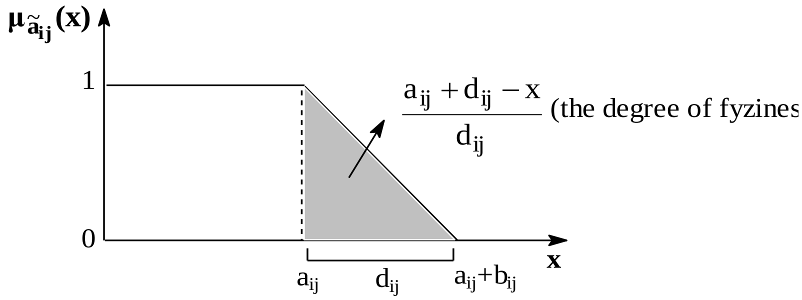

Fig. 2Membership function of $\mu_{\tilde{a}_{ij}}(x)$.

Also:

$$\tilde{b}_i(x) = \{(x, \mu_{\tilde{b}_i}(x)) \mid x \in R\}$$

where:

$$\mu_{\tilde{b}_i}(x) = \begin{cases} 1, & if \quad x < b_i \\ \dfrac{b_j + p_i - x}{p_i}, & if \quad b_i \le x \le b_i + p_i \\ 0, & if \quad x > b_i + p_i \end{cases}$$

See Fig.4.

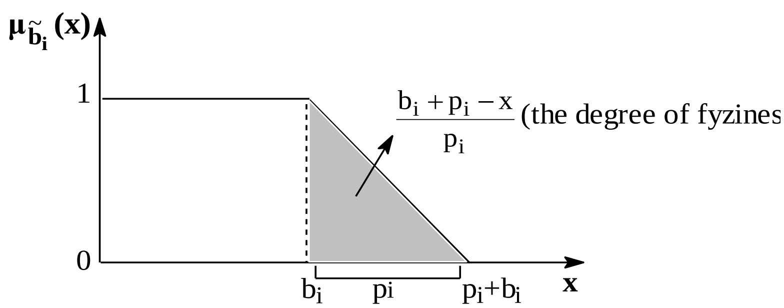

Fig.3The membership functionof $\mu_{\tilde{b}_i}(x)$.

Note that if any $\tilde{a}_{ij}$ and $\tilde{b}_i$ is negative, say $-\tilde{a}_{ij}$, then:

$$-\tilde{a}_{ij} = \{(x, \mu_{-\tilde{a}_{ij}}(x)) \mid x \in R\}$$

where

$$\mu_{-\tilde{a}_{ij}}(x) = \begin{cases} 1, & if \quad x < -(a_{ij} + d_{ij}) \\ \dfrac{x + (a_{ij} + d_{ij})}{d_{ij}}, & if \quad -(a_{ij} + d_{ij}) \le x \le -a_{ij} \\ 0, & if \quad x > -a_{ij} \end{cases}$$

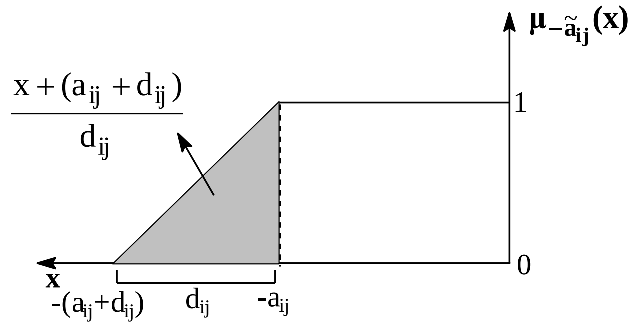

Fig. 4 Membership function of $\mu_{-\tilde{a}_{ij}}(x)$.

Similarly, use the same way in $\tilde{b}_i$.

2.Now, to fuzzify the i-th constraints $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le (\ge)\ \tilde{b}_i$, i = 1, 2, …, m

Let $\tilde{C}_i$ be the fuzzy set for i-th constraints, such that:

$\tilde{C}_i = \{(x,\ \mu_{\tilde{C}_i}(x)) \mid x \in R^n\}$

$\mu_{\tilde{C}_i}(x)$ can be defined by

$$\mu_{\tilde{C}_i}(x) = \begin{cases} 0, & if\ \ b_i < \sum_{j=1}^{n} a_{ij} x_j \\ \dfrac{b_i - \sum_{j=1}^{n} a_{ij} x_j}{\sum_{j=1}^{n} d_{ij} x_j + p_i}, & if\ \ \sum_{j=1}^{n} a_{ij} x_j \le b_i \le \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j + p_i \\ 1, & if\ \ b_i \ge \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j + p_i \end{cases}$$

Then we have the following crisp NLPP problem:
Max λ
Subject to:

$$\begin{aligned} &\bar{g}_1 : \lambda - \mu_{\tilde{M}}(x) \le 0 \\ &\bar{g}_2 : \lambda - \mu_{\tilde{C}_1}(x) \le 0 \\ &\vdots \\ &\bar{g}_{m+1} : \lambda - \mu_{\tilde{C}_m}(x) \le 0 \end{aligned} \tag{11}$$

Where x ≥ 0, 0 ≤λ≤ 1 and $x \in R^n$, which is equivalent to the problem (12):
Min (−λ)
Subject to:

$$\bar{g}_1 : \frac{f(x_j) - z_\ell}{z_u - z_\ell} - \lambda \geq 0$$

$$\bar{g}_2 : b_1 - [(a_{11} + \lambda d_{11})x_1 + \ldots + (a_{1n} + \lambda d_{1n})x_n] - \lambda p_1 \geq 0 \qquad (12)$$

$$\vdots$$

$$\bar{g}_{m+1} : b_m - [(a_{m1} + \lambda d_{m1})x_1 + \ldots + (a_{mn} + \lambda d_{mn})x_n] - \lambda p_m \geq 0$$

$\forall\, x_j = (x_1, x_2, \ldots, x_n) \geq 0$, $x_j \in R^n$ and $0 \leq \lambda \leq 1$.

The penalty function for problem (8) is given by:

$$\text{Min } \varphi(x_j, \lambda, \sigma) = -\lambda + \frac{1}{2}\sigma \sum_{i=1}^{m+1} [\text{Min}(0, \bar{g}_i)]^2, \; j = 1, 2, \ldots, n.$$

V. Numerical Example

Consider the following crisp nonlinear programming problem:

$\text{Min } z = x_1 + x_2 - x_1x_2$

Subject to:

$g_1: -8x_1 - 3x_2 \geq -6$ (13)

$g_2: 3x_1 + 6x_2 \geq 4$

$x_1, x_2 \geq 0$

The solution of the crisp problem by using the penalty function method:

$$\text{Min } \varphi(x_1, x_2, \sigma) = x_1 + x_2 - x_1x_2 + \frac{1}{2}\sigma \sum_{i=1}^{2} [\text{Min}(0, g_i)]^2$$

in algorithm (2.7.1), we get the following optimal results:

At $\sigma = 75\times10^4$ the penalty term $\frac{1}{2}\sigma \sum_{i=1}^{2} [\text{Min}(0, g_i)]^2$ equal to zero, for any given points $x_1 = 1.5$, $x_2 = 2$, we have:

$x_1^* = 0.16437888$, and $x_2^* = 0.58449170$,

Therefore $z^* = 0.6279249$ which satisfy the constraints:

$g_1^* = 2.931$ and $g_2^* = 8.684\times10^{-5}$

The fuzzy version of the above problems:

$\widetilde{\text{Min}}\; x_1 + x_2 - x_1x_2 = z$

Subject to:

$g_1: -\tilde{8}\, x_1 - \tilde{3}\, x_2 \geq -\tilde{6}$ (14)

$g_2: \tilde{3}\, x_1 + \tilde{6}\, x_2 \geq \tilde{4}$

$x_1, x_2 \geq .0$

1. Let $\tilde{M}$ be the fuzzy set of the objective function z, such that $\tilde{M} = \{(x, \mu_{\tilde{M}}(x)) \mid x \in R\}$, where $\mu_{\tilde{M}}(x)$ can be defined as:

$$\mu_{\tilde{M}}(x) = \begin{cases} 0, & if \quad x_1 + x_2 - x_1x_2 < z_\ell \\ \dfrac{x_1 + x_2 - x_1x_2 - z_\ell}{z_u - z_\ell}, & if \quad z_\ell \le x_1 + x_2 - x_1x_2 \le z_u \\ 1, & if \quad x_1 + x_2 - x_1x_2 > z_u \end{cases}$$

First, we find the chosen coefficients d_{ij} and p_i then we obtain :

$a_{ij} = \begin{bmatrix} -8 & -3 \\ 3 & 6 \end{bmatrix}$, $d_{ij} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, then $a_{ij} + d_{ij} = \begin{bmatrix} -7 & -1 \\ 7 & 9 \end{bmatrix}$

$b_i = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$, $p_i = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, then $b_i + p_i = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$, where i = 1, 2 and j = 1, 2.

and the lower and upper bounds z_λ and z_u for the optimal values can be found by solving four crisp NLPP as follows:

z_1= Min $x_1 + x_2 - x_1x_2$

Subject to:

g_1: $-8x_1 - 3x_2 \ge -6$ (15)

g_2: $3x_1 + 6x_2 \ge 4$

$x_1, x_2 \ge 0$.

z_1 have the same solution of z in (13).

z_2= Min $x_1 + x_2 - x_1x_2$

Subject to:

g_1: $-8x_1 - 3x_2 \ge -5$

g_2: $3x_1 + 6x_2 \ge 8$ (16)

$x_1, x_2 \ge 0$.

Using algorithm in section IV, we get the following optimal results:

At $\sigma = 1\times10^7$ the penalty term $\frac{1}{2}\sigma\sum_{i=1}^{2}[\text{Min}(0,g_i)]^2$ equal to zero, we get the results:

$x_1^* = 0.15383577$ and $x_2^* = 1.25643698$,

Therefore z_2= 1.2169878 , $g_1^* = 2.9\times10^{-6}$ and $g_2^* = 1.291\times10^{-4}$

z_3= Min $x_1 + x_2 - x_1x_2$

Subject to:

g_1: $-7x_1 - x_2 \ge -5$ (17)

g_2: $7x_1 + 9x_2 \ge 8$

$x_1, x_2 \geq 0$.

By using the algorithm method in section IV, the penalty term $\frac{1}{2}\sigma\sum_{i=1}^{2}[\text{Min}(0,g_i)]^2$ equal to zero at $\sigma = 1\times10^5$, so we get the optimal results:

$x_1^* = 0.43310236$ and $x_2^* = 0.55203368$.

Therefore z_3=0.74604895 that satisfies the constraintsg_1=1.416 and$g_2 = 1.963\times10^{-3}$

Finally, $z_4 = \text{Min } x_1 + x_2 - x_1x_2$

Subject to:

$g_1 : -7x_1 - x_2 \geq -6$ (18)

$g_2 : 7x_1 + 9x_2 \geq 4$

$x_1, x_2 \geq 0$.

Also, by using the algorithm section IV, the penalty term $\frac{1}{2}\sigma\sum_{i=1}^{2}[\text{Min}(0,g_i)]^2$ equal to zero at $\sigma = 1\times10^8$, with the optimal results:

$x_1^* = 0.42857293$,and $x_2^* = 0.14166517$,

Therefore z_4= 0.33426089That satisfy the constraintsg_1= 3.674,and g_2= 3.999.

Hence z_4= Max $\{z_1, z_2, z_3, z_4\}$=1.21698780 and z_λ= Min $\{z_1, z_2, z_3, z_4\}$ = 0.33426089. Therefore:

$$\mu_{\tilde{M}}(x) = \begin{cases} 1, & \text{if } 1.216980 < x_1 + x_2 - x_1x_2 \\ \dfrac{x_1 + x_2 - x_1x_2 - 0.33426089}{1.21698780 - 0.33426089}, & \text{if } 0.33426089 \leq x_1 + x_2 - x_1x_2 \leq 1.21698780 \\ 0, & \text{if } x_1 + x_2 - x_1x_2 < 0.33426089 \end{cases}$$

2-also let $\tilde{C}_1$ be the fuzzy set for the first constraint g_1, such that:

$$\mu_{\tilde{C}1}(x) = \begin{cases} 0, & \text{if } -6 < -8x_1 - 3x_2 \\ \dfrac{-6 - (-8x_1 - 3x_2)}{4x_1 + 3x_2 + 4}, & \text{if } -8x_1 - 3x_2 \leq -6 \leq -7x_1 - 1x_2 + 1 \\ 1, & \text{if } 4 \geq 7x_1 + 9x_2 + 4 \end{cases}$$

And let $\tilde{C}_2$ be the set for the second constraint g_2, such that:

$$\mu_{\tilde{C}_2}(x) = \begin{cases} 0, & \text{if } \; 4 < 3x_1 + 6x_2 \\ \dfrac{4-(3x_1+6x_2)}{4x_1+3x_2+4}, & \text{if } \; 3x_1 + 6x_2 \le 4 \le 7x_1 + 9x_2 + 4 \\ 1, & \text{if } \; 4 \ge 7x_1 + 9x_2 + 4 \end{cases}$$

1.By using fuzzy decision making as in problem (12), we have the following crisp NLPP:

Max λ

Subject to:

$$\bar{g}_1 : \lambda - \mu_{\tilde{M}}(x) \le 0$$

$$\bar{g}_2 : \lambda - \mu_{\tilde{C}_1}(x) \le 0$$

$$\bar{g}_3 : \lambda - \mu_{\tilde{C}_m}(x) \le 0$$

Where $x \ge 0$, $0 \le \lambda \le 1$ and $x \in R^2$, which is equivalent to the problem below:

Min $(-\lambda)$

Subject to:

$$\bar{g}_1 : \frac{x_1 + x_2 - x_1 x_2 - 0.33426098}{1.2169878 - 0.33426098} - \lambda \ge 0$$

$$\bar{g}_2 : \frac{-6x_1 + 8x_2 + 3x_2}{x_1 + 2x_2 + 1} - \lambda \ge 0 \qquad (19)$$

$$\bar{g}_3 : \frac{4 - 3x_1 - 6x_2}{4x_1 + 3x_2 + 4} - \lambda \ge 0$$

$x_1, x_2 \ge 0$, $0 \le \lambda \le 1$

The penalty function of the above problem is:

$$\text{Min } \varphi(x_1, x_2, \lambda, \sigma) = -\lambda + \frac{1}{2}\sigma \sum_{i=1}^{3} [\text{Min}(0, \bar{g}_i)]^2$$

Table I Result of problem (13)

σ	x_1	x_2	λ
10	1.5	2	0.5
100	0.84819258	–0.06900019	0.34780614
1000	0.80510498	–0.00681909	0.23532029
1×10^4	0.80037352	–0.00049138	0.22322137
1×10^5	0.80228107	–0.00009775	0.22153426
25×10^4	0.79977446	–00000580	0.22114719
5×10^5	0.79977446	–00000580	0.22114719
75×10^5	0.79975981	–0.00000066	0.22115274
1×10^6	0.79975981	–0.00000066	0.22115274
25×10^5	0.79975981	–0.00000066	0.22115274
5×10^6	0.79975981	–0.00000066	0.22115274
75×10^5	0.79975981	–0.00000066	0.22115274

1×10^{7}	0.79975981	–0.00000047	0.22115274
25×10^{6}	0.79975981	–0.00000047	0.22115274
5×10^{7}	0.79975981	–0.00000047	0.22115274
75×10^{6}	0.79974898	0.00001033	0.2114567
1×10^{8}	0.79974898	0.00001033	0.2114567

At $\sigma= 1\times10^{8}$ the penalty term $\frac{1}{2}\sigma\sum_{i=1}^{2}[\mathrm{Min}(0,g_i)]^2$ equal to zero, so we have the optimal solutions:

$X_1^* = 0.79974898, X_2^* = 0.00001033$ and $\lambda^* = 0.2114567$,

that satisfiesapproximately the constraints $g_1= 3.061\times10^{-1}$, $g_2= 6.42\times10^{-6}$ and $g_3= 1.202\times10^{-3}$ and $z_{Af}= 0.79975104$, such that $z_\lambda < z_{Af} < z_u$, where$z_{Af}$is the solution after the fuzziness .

VI. Conclusion

In this work, the a numerical method of fuzzy nonlinear programming problem is presented. Furthermore, it is proposed that the results solutionof fuzzy optimization is a generalization of the solution of the crisp optimization problem. In our work, the penalty function mixed with Nelder and Mend's algorithm have been successfully employed to solve numerical problems in fuzzy environment with fuzzy objective function and fuzzy technological constraints. The numerical results of our proposed method satisfied the fuzzy set theory properties.

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